A-436 (E/H) HIGHER MATHEMATICS 2017

Time : 3 Hours] **Class : 12th** **[M. M. : 100**

Instructions:

(i) All questions are compulsory.

(ii) Read instructions carefully of the question paper and then write answers of the questions.

(iii) Question paper has two sections-Section 'A' and Section 'B'.

(iv) In the Section 'A' Question Nos.1 to 5 are objective type. Each question carries 5 marks.

(v) In the Section 'B' Question Nos. 6 to 26 has Internal option.

(vi) Q.Nos 6 to 10 carry 2 marks each.

(vii) Q. Nos. 11 to 14 carry 3 marks each.

(viii) Q. Nos. 15 to 21 carry 4 marks each.

(ix) Q. Nos. 22 to 26 carry 5 marks each.

Section 'A'

Q. 1. Choose the correct options: $5 \times 1 = 5$

(i) if $\frac{2x+3}{x^2+5c+6} = \frac{A}{x+2} + \frac{B}{x+3}$ then the value of A + B is :

(a) 3 (b) 2

(c) 5 (d) 4

(ii) The value of $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$ is:

(a) $\cos^{-1} x$ (b) $\cot^{-1} x$

(c) $\sin^{-1}\frac{1}{x}$ (d) $\sin^{-1} x$

(iii) The value of $\int_1^3 x^2 dx$ is:

(a) $\frac{26}{3}$ (b) $\frac{28}{3}$

(c) $\frac{25}{3}$ (d) $\frac{8}{3}$

(iv) The unit vector in the direction of $\vec{a}$ is:

(a) $\frac{\vec{a}}{|\vec{a}|}$ (b) $\vec{a}\,|\vec{a}|$

(c) a^2 (d) $\hat{i}$

(v) Differential coefficient of log sin x is:

(a) cos x (b) tan x

(c) cosec x (d) cot x

Q.2. Write True/ False in the following statements: $5 \times 1 = 5$

(i) The value of correlation coefficient lies between - 2 to + 2

(ii) If regression coefficient are 0.8 and 0.2 then the value of correlation coefficient is + 0.4

(iii) The function $f(x) = 5x + 2$ is increasing on set of real numbers.

(iv) If the position vector of centre of sphere is $\vec{c}$ and radius is a then the vector equartion of sphere is $|\vec{r} + \vec{c}| = a$.

(v) Two vectros $\vec{a}$ and $\vec{b}$ are perpendicular if $\vec{a} \cdot \vec{b} = 0$

Q.3. Fill in the blanks: $5 \times 1 = 5$

(i) The perpendicular distance of a point (5, 12, 13) from Y-axis is.....

(ii) The equation of the plane passing through the origin (0, 0, 0) is .....

(iii) The intercept cut by the plane $2x + y - z = 5$ on X-axis is .....

(iv) The centre of the sphere $x^2 + y^2 + z^2 + 3x + 5y + 2z = 0$ is .....

(v) The differential coefficient of sin 3x with respect to 3x is.....

Q.4. Give answer in one word/sentence: $5 \times 1 = 5$

(i) In Newton-Raphson's method write formula for finding square root of the number N.

(ii) Write trapezoidal rule formula in numerical methods.

(iii) Write cube root of 10by Newton-Raphson's method after first iteration.

(iv) In which interval does the root of equation $x^3 + x - 3 = 0$ lie.

(v) Write the coefficient of y with odd subscripts in Simpson's Rule.

Q.5. Match the correct pair. $5 \times 1 = 5$

'A'	'B'
(a) $\int \text{cosec } x dx$	(i) $\sec^{-1} x + c$
(b) $\int \frac{dx}{x\sqrt{x^2-1}}$	(ii) $\frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \sin^{-1}\frac{x}{a}\right] + c$
(c) $\int \sqrt{a^2 - x^2} dx$	(iii) $\log\left[x + \sqrt{x^2 - a^2}\right] + c$
(d) $\int \frac{dx}{\sqrt{a^2 - x^2}}$	(iv) $\log \tan \frac{x}{a} + c$
(e) $\int \frac{dx}{a^2 + x^2}$	(v) $\sin^{-1}\frac{x}{a} + c$
	(vi) $\frac{1}{a}\tan^{-1}\frac{x}{a} + c$

Section B

Q.6. Prove that the vectors $\vec{a} = 5\hat{i} + 15\hat{j}$ and $\vec{b} = 3\hat{i} + 9\hat{j}$ are parallel. 2

(OR) If the position vectors of the points A and B are $7\hat{i} + 3\hat{j} + \hat{k}$ and $2\hat{i} + 5\hat{j} + 4\hat{k}$ respectively, then find the magnitude of $\vec{AB}$.

Q.7. The displacement of a particle by the force $\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$ is $\vec{d} = 3\hat{i} + 2\hat{j} - 5\hat{k}$, then find the work done by the force. 2

(OR) If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} + \hat{k}$, then find the value of $\vec{a} \times \vec{b}$.

Q. 8. For what value of λ the planes $\vec{r}.(2\hat{i} + \lambda\hat{j} - 3\hat{k}) = 2$ and $\vec{r}.(\lambda\hat{i} - 3\hat{j} + \hat{k}) = 5$ are perpendicular to each other.

(OR) Write the formula to tind the shortest distance between two straight lines whose vector equation are

$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$

and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$

where λ and μ are scalars.

Q. 9. Find the value of $\int \frac{\cos(\log x)}{x} dx$. 2

(OR) Prove that $\int \sec x \, dx = \log(\sec x + \tan x) + c$.

Q. 10. find the value of $\int x e^x dx$. 2

(OR) find the value of $\int x \tan^2 x \, dx$.

Q. 11. Find the distance between the parallel planes $2x - 2y + z + 3 = 0$ and $4x - 4y + 2z + 5 = 0$. 3

(OR) In which ratio does the YZ plane divide the line joining the points (-2, 4, 7) and (3, -5, 8).

Q. 12. Prove that the symmetrical form of equation $x = ay + b$, $z = cy + d$ is $\frac{x - b}{a} = \frac{y}{1} = \frac{z - d}{c}$. 3

(OR) Find the equation of the sphere whose centre is (3, 2, 1) and radius is 5.

Q. 13. Prove by vector method that the angle in a semicircle is right angle. 3

(OR) Prove that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$

Q. 14. Find the vector equation of the spher where the points (2, 3, 5) and (4, 9, -3) are extremities of its diameter. 3

(OR) Find the angle between the planes $\vec{r}.(2\hat{i} + 3\hat{j} + 4\hat{k}) = 1$ and $\vec{r}.(\hat{i} + \hat{j}) = 4$.

Q. 15. Resolve $\frac{13x+18}{2x^2+5x+3}$ into partial fractions. 4

(OR) Resolve $\frac{8x^2-9}{(3x+2)(x^2+5)}$ into partial fractions.

Q. 16. Prove that $\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{3}{5}=\tan^{-1}\frac{27}{11}$. 4

(OR) Express $\sin\left[2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right]$ in simplest form.

Q. 17. Find differential coefficient of $\sqrt{\tan\sqrt{x}}$. 4

(OR) If $y=\cot^{-1}\left[\frac{\sqrt{1+x^2}+1}{x}\right]$, then find the value of $\frac{dy}{dx}$.

Q. 18. Differentiate $(\cos x)^{\cos x}$ with respect to x. 4

(OR) if $y=\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos+.....\infty}}}$ then prove that

$(1-2y)\frac{dy}{dx}=\sin x$

Q. 19. The side of a square sheet of metal is increasing at the rate of 5cm/minute. At what are its area is increasing when the side is 20cm long? 4

(OR) Find the maximum value of $2x^3-24x+107$ in the interval [1, 3].

Q. 20. Calculate the correlation coefficient between x and y for the following data: 4

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

(OR) Calculate cov (X,Y) between two variables x and y where:

$\sum x_i = 15, \sum y_i = 36, \sum x_i y_i = 110, n = 5$

Q. 21. Prove that correlation coefficient is the geometric mean of the regression coefficiens. 4

(OR) Calculate regression coefficient byx and [illegible] for variables x and y for the following data:

$\sum x = 24,\ \sum y = 44,\ xy = 306,\ \sum x^2 = 164,\ \sum y^2 = 574,\ n = 4$

Q. 22. Find the equation of the plane passing through the point (4, 5, 1), (0, -1, -1) and (-4, 4, 4). 5

(OR) Prove that the lines $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ are coplanar. Find the point of intersection of these lines.

Q. 23. If $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$ then prove that $f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$.

(OR) Find the value of $\lim_{x \to o} \frac{\tan x - \sin x}{x^3}$.

Q. 24. Find the value of $\int \frac{dx}{5 + 4\sin x}$. 5

(OR) Prove that $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}} dx = \frac{\pi}{4}$.

25. Solve the differenatial equation $(x-1)\frac{dy}{dx} = 2x^3y$. 5

(OR) Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + 5xy + 4y^2}{x^2}$.

Q. 26. A card is drawn at random from a well shuffled pack of of 52 cards. Find the probability thaty it is neither an ace nor a king. 5

(OR) A fair coin is tossed six times. What is the probability of getting at least three heads.?

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