

MATHEMATICS & STATISTICS**Time : 3 Hrs.****July - 2016****Marks : 80****SECTION - I**

Q.1.(A) Select and write the correct answer from the given alternatives in each of the following sub-questions: (6)

(1) Inverse of the statement pattern $(p \vee q) \rightarrow (p \wedge q)$ is (2)

(a) $(p \wedge q) \rightarrow (p \vee q)$

(b) $\sim (p \vee q) \rightarrow (p \wedge q)$

(c) $(\sim p \vee \sim q) \rightarrow (\sim p \wedge \sim q)$

(d) $(\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$

(2) If the vectors $2\hat{i} - q\hat{j} + 3\hat{k}$ and $4\hat{i} - 5\hat{j} + 6\hat{k}$ are collinear, then value of q is (2)

(a) 5

(b) 10

(c) $\frac{5}{2}$

(d) $\frac{5}{4}$

(3) If in $\triangle ABC$ with usual notations $a = 18$, $b = 24$, $c = 30$ then $\sin \frac{A}{2}$ is equal to (2)

(a) $\frac{1}{\sqrt{5}}$

(b) $\frac{1}{\sqrt{10}}$

(c) $\frac{1}{\sqrt{15}}$

(d) $\frac{1}{2\sqrt{5}}$

(B) Attempt any THREE of the following: (6)

(1) Find the angle between the lines (2)

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) \text{ and } \vec{r} = 5\hat{i} - 2\hat{k} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

(2) If p, q, r are the statements with truth value T, F, T, respectively then find the truth value of $(r \wedge q) \leftrightarrow \sim p$. (2)

(3) If $A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$ then find A^{-1} by adjoint method. (2)

(4) By vector method show that the quadrilateral with vertices A (1, 2, -1), B (8, -3, -4), C (5, -4, 1), D (-2, 1, 4) is a parallelogram. (2)

(5) Find the general solution of the equation $\sin x = \tan x$. (2)

Q.2.(A) Attempt any TWO of the following: (6)

(1) Find the joint equation of pair of lines passing through the origin and perpendicular to the lines represented by $ax^2 + 2hxy + by^2 = 0$ (3)

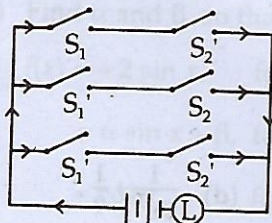
(2) Find the principal value of $\sin^{-1} \left(\frac{-1}{\sqrt{2}} \right)$ (3)

(3) Find the cartesian form of the equation of the plane (3)

$$\vec{r} = (\hat{i} + \hat{j}) + s(\hat{i} - \hat{j} + 2\hat{k}) + t(\hat{i} + 2\hat{j} + \hat{k})$$

(B) Attempt any TWO of the following: (8)

(1) Simplify the following circuit so that new circuit has minimum number of switches. Also draw simplified circuit. (4)



- (2) A line makes angles of measures 45° and 60° with positive direction of y and z axes respectively. Find the d.c.s. of the line and also find the vector of magnitude 5 along the direction of line. (4)

- (3) Maximize: $z = 3x + 5y$ (4)

Subject to : $x + 4y \leq 24$

$3x + y \leq 21$

$x + y \leq 9$

$x \geq 0, y \geq 0$

Q.3.(A) Attempt any TWO of the following: (6)

- (1) Find the shortest distance between the lines: (3)

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

- (2) Show that the points $(1, -1, 3)$ and $(3, 4, 3)$ are equidistant from the plane $5x + 2y - 7z + 8 = 0$ (3)

- (3) In any triangle ABC with usual notations prove $c = a \cos B + b \cos A$. (3)

(B) Attempt any TWO of the following: (8)

- (1) Find p and k if the equation $px^2 - 8xy + 3y^2 + 14x + 2y + k = 0$ represents a pair of perpendicular lines. (4)

- (2) The cost of 4 dozen pencils, 3 dozen pens and 2 dozen erasers is ₹ 60. The cost of 2 dozen pencils, 4 dozen pens and 6 dozen erasers is ₹ 90 whereas the cost of 6 dozen pencils, 2 dozen pens and 3 dozen erasers is ₹ 70. Find the cost of each item per dozen by using matrices. (4)

- (3) Prove that the volume of the parallelopiped with coterminus edges as $\vec{a}, \vec{b}, \vec{c}$ is $[\vec{a} \vec{b} \vec{c}]$ and hence find the volume of the parallelopiped with its coterminus edges $2\hat{i} + 5\hat{j} - 4\hat{k}$, $5\hat{i} + 7\hat{j} + 5\hat{k}$, and $4\hat{i} + 5\hat{j} - 2\hat{k}$. (4)

SECTION - II

Q.4.(A) Select and write the correct answer from the given alternatives in each of the following sub-questions: (6)

- (1) The order and degree of the differential equation (2)

$$\left[1 + \left(\frac{dy}{dx} \right)^3 \right]^{\frac{7}{3}} = 7 \left(\frac{d^2y}{dx^2} \right) \text{ are respectively.}$$

(a) 2, 3

(b) 3, 2

(c) 2, 2

(d) 3, 3

- (2) $\int_4^9 \frac{1}{\sqrt{x}} dx = \dots\dots\dots$ (2)

(a) 1

(b) -2

(c) 2

(d) -1

- (3) If the p.d.f. of a continuous random variable X is given as (2)

$$f(x) = \frac{x^2}{3} \quad \text{for } -1 < x < 2$$

$$= 0 \quad \text{otherwise.}$$

then c.d.f. of X is

(a) $\frac{x^3}{9} + \frac{1}{9}$

(b) $\frac{x^3}{9} - \frac{1}{9}$

(c) $\frac{x^2}{4} + \frac{1}{4}$

(d) $\frac{1}{9x^3} + \frac{1}{9}$

(B) Attempt any THREE of the following:

- (1) If $y = \sec \sqrt{x}$, find $\frac{dy}{dx}$. (2)
- (2) Evaluate: $\int \frac{(x+1)}{(x+2)(x+3)} dx$ (2)
- (3) Find the area of the region lying in the first quadrant bounded by the curve $y^2 = 4x$, X axis and the lines $x = 1$, $x = 4$. (2)
- (4) Solve the differential equation: $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$ (2)
- (5) Given $X \sim B(n, p)$ if $E(X) = 6$, $\text{Var}(X) = 4.2$, find the value of n and p . (2)

Q.5. (A) Attempt any TWO of the following:

- (1) If the function $f(x) = \frac{(4^{\sin x} - 1)^2}{x \cdot \log(1+2x)}$, for $x \neq 0$ is continuous at $x = 0$, find $f(0)$ (3)
- (2) Evaluate: $\int \frac{1}{3+2\sin x + \cos x} dx$ (3)
- (3) If $y = f(x)$ is differentiable function of x such that inverse function $x = f^{-1}(y)$ exists then prove that x is a differentiable function of y and $\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$, where $\frac{dy}{dx} \neq 0$ (3)

(B) Attempt any TWO of the following:

- (1) A point source of light is hung 30 feet directly above a straight horizontal path on which a man of 6 feet in height is walking. How fast is the man's shadow lengthening and how fast the tip of shadow is moving when he is walking away from the light at the rate of 100ft/min? (4)
- (2) The p.m.f. for $X =$ number of major defects in a randomly selected appliance of a certain type is (4)

$X = x$	0	1	2	3	4
$P(x)$	0.08	0.15	0.45	0.27	0.05

Find the expected value and variance of X .

- (3) Prove that $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ hence evaluate $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$. (4)

Q.6. (A) Attempt any TWO of the following:

- (1) If $y = e^{\tan x} + (\log x)^{\tan x}$ then find $\frac{dy}{dx}$. (3)
- (2) If the probability that a fluorescent light has a useful life of at least 800 hours is 0.9, find the probabilities that among 20 such lights at least 2 will not have a useful life of at least 800 hours. [Given : $(0.9)^{19} = 0.1348$] (3)
- (3) Find α and β , so that the function $f(x)$ is defined by (3)

$$\begin{aligned}
 f(x) &= -2 \sin x, & \text{for } -\pi \leq x \leq -\frac{\pi}{2} \\
 &= \alpha \sin x + \beta, & \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2} \\
 &= \cos x, & \text{for } \frac{\pi}{2} \leq x \leq \pi \text{ is continuous on } [-\pi, \pi]
 \end{aligned}$$

(B) Attempt any TWO of the following: (8)

(1) Find the equation of a curve passing through the point (0, 2), given that the sum of the coordinates of any point on the curve exceeds the slope of the tangent to the curve at that point by 5. (4)

(2) If u and v are two functions of x then prove that: (4)

$$\int uv \, dx = u \int v \, dx - \int \left[\frac{du}{dx} \int v \, dx \right] dx$$

Hence evaluate $\int x e^x dx$

(3) Find the approximate value of $\log_{10}(1016)$ given that $\log_{10} e = 0.4343$. (4)

