MATHEMATICS & STATISTICS

Time : 3 Hrs. **July - 2016** **Marks : 80**

SECTION - I

Q.1. Attempt any SIX of the following: **(12)**

(i) Evaluate: $\int \frac{e^{3x}}{e^{3x}+1}\,dx$

(ii) The price P for demand D is given as

$P = 183 + 120D - 3D^2$; find D for which price is increasing.

(iii) Write the truth value of the negation of each of the following statements:

(a) The Sun sets in the East. (b) $\cos^2\theta + \sin^2\theta = 1$, for all $\theta \in R$.

(iv) Simplify the following: $\left\{3\begin{bmatrix}1 & 2 & 0\\ 0 & -1 & 3\end{bmatrix} - \begin{bmatrix}1 & 5 & -2\\ -3 & -4 & 4\end{bmatrix}\right\}\begin{bmatrix}1\\2\\1\end{bmatrix}$

(v) Examine the continuity of f at $x = 1$, if

$f(x) = 5x - 3$, for $0 \le x \le 1$

$= x^2 + 1$, for $1 \le x \le 2$

(vi) Find $\frac{dy}{dx}$, if $y = x^{x^3}$

(vii) If $A = \begin{bmatrix}1 & 2 & 3\\ 2 & a & 2\\ 5 & 7 & 3\end{bmatrix}$ is a singular matrix, find the value of 'a'.

(viii) Evaluate: $\int \frac{1}{\sqrt{x^2-4x+2}}\,dx$

Q.2. (A) Attempt any TWO of the following: **(6)**

(i) Show that the following statement pattern is contingency:

$(\sim p \vee q) \rightarrow [p \wedge (q \vee \sim q)]$

(ii) If $f(x) = \frac{e^{2x}-1}{ax}$, for $x < 0, a \neq 0$

$= 1$, for $x = 0$

$= \frac{\log(1+7x)}{bx}$, for $x > 0, b \neq 0$

is continuous at $x = 0$, then find a and b.

(iii) If $x^y = e^{x-y}$, then show that

$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

(B) Attempt any TWO of the following: **(8)**

(i) Evaluate: $\int_0^1 x \cdot \tan^{-1} x\,dx$

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(ii) If $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, verify that

$A\,(\text{adj } A) = (\text{adj } A)A = |A| \cdot I$

(iii) A manufacturer can sell x items at a price of ₹ $(280 - x)$ each. The cost of producing x items is ₹ $(x^2 + 40x + 35)$. Find the number of items to be sold so that the manufacturer can make maximum profit.

Q.3. (A) Attempt any TWO of the following: **(6)**

(i) Find k, if the function f is continuous at $x = 0$, where

$$f(x) = \frac{(e^x - 1)(\sin x)}{x^2}, \quad x \neq 0$$
$$= k \quad , \quad x = 0$$

(ii) Differentiate $\log(1 + x^2)$ w.r.t. $\cot^{-1}x$

(iii) Using the Venn diagram, examine the logical equivalence of the following statements:

(a) Some politicians are actors. (b) There are politicians who are actors.

(c) There are politicians who are not actors.

(B) Attempt any TWO of the following: **(8)**

(i) Find the volume of the solid generated by the complete revolution of the ellipse $\frac{x^2}{36} + \frac{y^2}{25} = 1$ about Y axis.

(ii) Evaluate: $\int \frac{x^2}{x^4 + 5x^2 + 6} dx$

(iii) The total cost of manufacturing x articles is $c = 47x + 300x^2 - x^4$. Find x, for which average cost is (a) increasing (b) decreasing.

SECTION - II

Q.4. Attempt any SIX of the following: **(12)**

(i) Let X = amount of time for which a book is taken out of a college library by a randomly selected student and suppose X has p.d.f.

$$f(x) = \begin{cases} 0.5x, & 0 \le x \le 2 \\ 0 & , \text{ otherwise} \end{cases}$$

Calculate (a) $P(X \le 1)$ (b) $P(0.5 \le X \le 1.5)$

(ii) If the correlation coefficient between X and Y is 0.8, what is the correlation coefficient between:

(a) $\frac{X}{2}$ and Y (b) $\frac{X-5}{7}$ and $\frac{Y-3}{8}$

(iii) Raghu, Madhu and Ramu started a business in a partnership by investing ₹ 60,000, ₹ 40,000 and ₹ 75,000 respectively. At the end of the year they found that they have incurred a loss of ₹ 24,500. Find that how much loss each one had to bear.

(iv) If the rank correlation coefficient is 0.6 and the sum of squares of differences of ranks is 66, then find the number of pairs of observations.

(v) For an immediate annuity paid for 3 years with interest compounded at 10% p.a. its present value is ₹ 10,000. What is its accumulated value after 3 years? [Given: $(1.1)^3 = 1.331$].

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(vi) Calculate CDR for district A and B.

Age group (in years)	District A		District B	
	Population	No. of Deaths	Population	No. of Deaths
0 – 10	1000	18	3000	70
10 – 55	3000	32	7000	50
Above 55	2000	41	1000	24

(vii) Compute Age-Specific Death rate for the following data:

Age group (in years)	Population (in thousands)	No. of Deaths
Below 5	15	360
5 – 30	20	400
Above 30	10	280

(viii) The ratio of prices of two houses was 2 : 3. Two years later when price of first house has increased by 30% and that of the second by ₹ 90,000 the ratio of prices becomes 5 : 7. Find the original prices of two houses.

Q.5. (A) Attempt any TWO of the following: **(6)**

(i) The income of an agent remains unchanged though the rate of commission is increased from 5% to 6.25%. Find the percentage reduction in the value of business.

(ii) A new treatment for baldness is known to be effective in 70% of the cases treated. Four bald members from different families are treated. Find the probability that
(a) exactly two members are successfully treated.
(b) at least one member is successfully treated.

(iii) Find the graphical solution for following system of linear inequalities:
$\frac{x_1}{60} + \frac{x_2}{90} \le 1, 5x_1 + 8x_2 \le 600, x_1 \ge 0, x_2 \ge 0$

(B) Attempt any TWO of the following: **(8)**

(i) Bring out the inconsistency, if any in the following:

(a) $b_{YX} + b_{XY} = 1.30$ and $r = 0.75$

(b) $b_{YX} = b_{XY} = 1.50$ and $r = -0.9$

(c) $b_{YX} = 1.9$ and $b_{XY} = -0.25$

(d) $b_{YX} = 2.6$ and $b_{XY} = \frac{1}{2.6}$

(ii) A pharmaceutical company has four branches, one each at city A, B, C, D. A branch manager is to be appointed one at each city, out of four candidates P, Q, R and S. The monthly business depending upon the city and the effectiveness of the branch manager in that city is given below:

	City			
	A	B	C	D
	Monthly business (₹ lakh)			
P	10	10	8	8
Q	12	15	10	9
R	11	16	12	7
S	15	13	15	11

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Which manager should be appointed at which city so as to get the maximum total monthly business?

(iii) Fill in the blanks which are marked with a question mark in the following extract from the life table:

x	l_x	d_x	q_x	p_x	L_x	T_x
20	88230	?	?	?	?	?
21	79473					3205552

Q.6. (A) Attempt any TWO of the following: **(6)**

(i) For a bivariate data,

$\bar{x} = 53, \bar{y} = 28, b_{YX} = -1.5$ and $b_{XY} = -0.2$

Find

(a) Correlation coefficient between X and Y.

(b) Estimate of Y for X = 50

(c) Estimate of X for Y = 25

(ii) If X has a Poisson distribution with variance 2, find

(a) $P(X = 4)$ (b) $P(X \leq 4)$ (c) Mean of X [Use $e^{-2} = 0.1353$]

(iii) There are 5 jobs each of which is to be processed through three machines. A, B and C in the order ABC. Processing times in hours are as shown in the following table:

Jobs	1	2	3	4	5
A	3	8	7	5	4
B	4	5	1	2	3
C	7	9	5	6	10

Determine the optimum sequence for the five jobs and the minimum elapsed time.

(B) Attempt any TWO of the following: **(8)**

(i) A bill of ₹ 7,500 was discounted for ₹ 7,290 at a bank on 28th October, 2006. If the rate of interest was 14% p.a., what is the legal due date?

(ii) The following data gives the marks of 20 students in Mathematics (X) and Statistics (Y), each out of 10, expressed as (X, Y). Construct ungrouped frequency distribution considering single number as a class. Also prepare marginal distributions.

(2, 7), (3, 8), (4, 9), (2, 8), (2, 8), (5, 6), (5, 7), (4, 9) (3, 8), (4, 8), (2, 9), (3, 8), (4, 8), (5, 6), (4, 7), (4, 7), (4, 6), (5, 6), (5, 7), (4, 6).

(iii) Minimise: $Z = 3x_1 + 2x_2$

Subject to constraints

$5x_1 + x_2 \geq 10;$

$2x_1 + 2x_2 \geq 12;$

$x_1 + 4x_2 \geq 12;$

$x_1, x_2 \geq 0$

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