

This Question Paper contains 20 printed pages

*Engineering paper*

**Sl.No.**

050 (E)

(MARCH/APRIL, 2015)

**Part - A : Time : 1 Hour / Marks : 50**

**Part - B : Time : 2 Hours / Marks : 50**

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Set No. of  
Question Paper:

09

### (Part - A)

*Time : 1 Hour*

*[Maximum Marks : 50]*

### Instructions :

- 1) There are 50 objective type (M.C.Q.) questions in Part - A and all questions are compulsory.
  - 2) The questions are serially numbered from 1 to 50 and each carries 1 mark.
  - 3) Read each question carefully, select proper alternative and answer in the O.M.R. sheet.
  - 4) The OMR sheet is given for answering the questions. The answer of each question is represented by (A) O, (B) O, (C) O, (D) O. Darken the circle ● of the correct answer with ball-pen.
  - 5) Rough work is to be done in the space provided for this purpose in the Test Booklet only.
  - 6) Set No. of Question Paper printed on the upper-most right side of the Question Paper is to be written in the column provided in the OMR sheet.
  - 7) Use of Simple Calculator and log table is allowed, if required.

$$1) \quad \lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a} = \underline{\hspace{2cm}}$$

- (A)  $-\sin a$       (B)  $-\sin x$   
 (C)  $\cos x$       (D)  $\cos a$

$$2) \quad \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{1 - \cos 2x} = \underline{\hspace{2cm}}$$



## Rough Work

## Rough Work

3)  $\lim_{x \rightarrow 16} \frac{x^{\frac{1}{4}} - 2}{\sqrt{x} - 4} = \underline{\hspace{2cm}}$

(A) 2

(B)  $-\frac{1}{2}$ (C)  $\frac{1}{4}$ 

(D) 1

4)  $\lim_{x \rightarrow 0} \frac{\tan x - 20x}{9x - \sin x} = \underline{\hspace{2cm}}$

(A)  $\frac{9}{20}$ (B)  $\frac{20}{9}$ (C)  $-\frac{19}{8}$ 

(D) 0

5)  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \sin x - 1}{\cos 2x} = \underline{\hspace{2cm}}$

(A)  $-\frac{1}{2}$ (B)  $\frac{1}{2}$ (C)  $\sqrt{2}$ (D)  $\frac{1}{\sqrt{2}}$

6) If  $f(x) = x^{100} + x^{99} + x^{98} + \dots + 1$  then  $f'(1) = \underline{\hspace{2cm}}$ .

- (A) 505
- (B) 5000
- (C) 5050
- (D) 5500

7) If  $f(x) = \cos^2 x$  then  $f'(\frac{\pi}{6}) = \underline{\hspace{2cm}}$ .

- (A)  $\frac{1}{2}$
- (B)  $\sqrt{3}$
- (C)  $\frac{\sqrt{3}}{2}$
- (D)  $-\frac{\sqrt{3}}{2}$

8)  $\frac{d}{dx} (\sin^2(x^4) + \cos^2(x^4)) = \underline{\hspace{2cm}}$ 

- (A) 0
- (B) 1
- (C) -1
- (D)  $2(\sin x^4 + \cos x^4)$

9)  $\frac{d}{dx} (\cot x^\circ) = \underline{\hspace{2cm}}$ 

- (A)  $-\frac{\pi}{180} \operatorname{cosec}^2 x^\circ$
- (B)  $-\operatorname{cosec}^2 x^\circ$
- (C)  $\frac{\pi}{180} \operatorname{cosec}^2 x^\circ$
- (D) 0

## Rough Work

10)  $\frac{d}{dx}(x|x|) = \text{_____}$  (where  $x < 0$ )

- (A) 0
- (B)  $2x$
- (C) -2
- (D)  $-2x$

11)  $P(n) : n^2 - n + 41$  is a prime is false for  $n = \text{_____}$

- |       |        |
|-------|--------|
| (A) 3 | (B) 1  |
| (C) 2 | (D) 41 |

12)  $\left[ i^{19} + \left( \frac{1}{i} \right)^{25} \right]^2 = \text{_____}$

- |            |           |
|------------|-----------|
| (A) -4     | (B) 4     |
| (C) $2i^2$ | (D) $-2i$ |

13) If  $x + 2yi = xi + y + 3$  then values of  $x$  and  $y$  are respectively  
 $\text{_____}$  and  $\text{_____}$ .

- |           |          |
|-----------|----------|
| (A) -6, 3 | (B) 6, 3 |
| (C) 4, 2  | (D) 3, 6 |

14) Find the multiplicative inverse of  $Z = 3 - 4i$ .

(A)  $\frac{3}{25} - \frac{4i}{25}$

(B)  $-\frac{3}{25} + \frac{4i}{25}$

(C)  $-\frac{3}{25} - \frac{4i}{25}$

(D)  $\frac{3}{25} + \frac{4i}{25}$

## Rough Work

15) If  $(a+ib)^2 = \frac{1+i}{1-i}$  then  $a^2 + b^2 = \underline{\hspace{2cm}}$ .

- |                |       |
|----------------|-------|
| (A) -1         | (B) 1 |
| (C) $\sqrt{2}$ | (D) 0 |

16) The middle term of  $\left(x^2 + \frac{1}{x^2}\right)^{2n}$  is  $\underline{\hspace{2cm}}$ , ( $x \neq 0$ )

- |                                      |
|--------------------------------------|
| (A) $\binom{2n}{n} \frac{1}{x^{2n}}$ |
| (B) $\binom{2n}{n} x^{2n}$           |
| (C) $\binom{2n}{n-1}$                |
| (D) $\binom{2n}{n}$                  |

17)  $\binom{n+1}{1} + \binom{n+1}{2} + \dots + \binom{n+1}{n} = \underline{\hspace{2cm}}$  where  $n \in \mathbb{N}$

- |                   |               |
|-------------------|---------------|
| (A) $2^{n+1} - 1$ | (B) $2^n - 1$ |
| (C) $2(2^n - 1)$  | (D) $2^n - 2$ |

18) The sum of co-efficients of  $a^{11} b^2$  and  $a^2 b^{11}$  in the expansion of  $(a+b)^{13}$  is  $\underline{\hspace{2cm}}$ .

- |                      |                       |
|----------------------|-----------------------|
| (A) $2\binom{13}{2}$ | (B) $\binom{13}{11}$  |
| (C) $\binom{13}{10}$ | (D) $2\binom{13}{10}$ |

19) The fifth term in expansion of  $(3x - y)^7$  is \_\_\_\_\_.

- (A)  $945 x^4 y^3$
- (B)  $945 x^3 y^4$
- (C)  $-945 x^3 y^4$
- (D)  $-2835 x^4 y^3$

20) The value of  $\sin\left(-\frac{50\pi}{3}\right)$  is \_\_\_\_\_.

- (A)  $-\frac{\sqrt{3}}{2}$
- (B)  $\frac{\sqrt{3}}{2}$
- (C)  $\frac{1}{2}$
- (D)  $-\frac{1}{2}$

21) The maximum value of function  $f(x) = 5 \cos x - 12 \sin x - 13$  is \_\_\_\_\_.

- (A) 13
- (B) 26
- (C) 0
- (D) -26

22) If  $A = 128$  and  $x = \sin A^\circ + \cos A^\circ$  then \_\_\_\_\_

- (A)  $x > 0$
- (B)  $x < 0$
- (C)  $x = 0$
- (D)  $x \geq 0$

23) Minimum value of  $f(x) = \sin^4 x + \cos^4 x$  is \_\_\_\_\_

- |       |                   |
|-------|-------------------|
| (A) 1 | (B) $\frac{1}{2}$ |
| (C) 0 | (D) -1            |

Rough Work

24) If  $\cos \alpha = -\frac{3}{5}$ ,  $\frac{\pi}{2} < \alpha < \pi$  then the value of  $\cos \frac{\alpha}{2}$  is \_\_\_\_\_.

- |                           |
|---------------------------|
| (A) $-\frac{1}{\sqrt{5}}$ |
| (B) $\frac{1}{\sqrt{5}}$  |
| (C) $\frac{2}{\sqrt{5}}$  |
| (D) $-\frac{3}{10}$       |

25) One of the root of  $8x^3 - 6x = 1$  is \_\_\_\_\_

- |                     |                     |
|---------------------|---------------------|
| (A) $\sin 70^\circ$ | (B) $\sin 20^\circ$ |
| (C) $\cos 80^\circ$ | (D) $\cos 10^\circ$ |

26) If  $\tan \theta = 3$  then  $\tan 4\theta =$  \_\_\_\_\_

- |                      |
|----------------------|
| (A) $-\frac{24}{7}$  |
| (B) $\frac{24}{7}$   |
| (C) $-\frac{24}{25}$ |
| (D) $\frac{24}{25}$  |

27) The value of  $\sin 18^\circ \cos 36^\circ + \sin 15^\circ \cos 15^\circ$  is \_\_\_\_\_

(A)  $\frac{1}{2}$

(B) 1

(C) 2

(D)  $\frac{1}{4}$

28) If  $\sin B = \frac{3}{5}$ ,  $0 < B < \frac{\pi}{2}$  then  $\sin 2B + \cos 2B =$  \_\_\_\_\_.

(A)  $\frac{24}{25}$

(B)  $\frac{31}{25}$

(C)  $\frac{7}{25}$

(D)  $\frac{17}{25}$

29) The general solution set of  $3 \cos \theta - 4 \sin \theta = 6$  is \_\_\_\_\_.

(A) R

(B)  $\emptyset$

(C)  $\left\{ \frac{\pi}{4} \right\}$

(D)  $\left\{ k\pi + \frac{\pi}{4} \mid k \in \mathbb{Z} \right\}$

30) The solution set of  $\sin \theta + \cos \theta = \sqrt{2}$  is \_\_\_\_\_.

(A)  $\left\{ k\pi - \frac{\pi}{4} \mid k \in \mathbb{Z} \right\}$

(B)  $\left\{ k\pi + \frac{\pi}{4} \mid k \in \mathbb{Z} \right\}$

(C)  $\left\{ 2k\pi - \frac{\pi}{4} \mid k \in \mathbb{Z} \right\}$

(D)  $\left\{ 2k\pi + \frac{\pi}{4} \mid k \in \mathbb{Z} \right\}$

31) In  $\triangle ABC$ ,  $A = \frac{\pi}{4}$ ,  $C = \frac{\pi}{3}$  then  $2b - a$  is \_\_\_\_\_.

(A)  $\sqrt{3} c$

(B)  $c$

(C)  $\sqrt{2} c$

(D)  $2c$

32) The number of solutions of the equation  $3 \cos^2 x - 7 \cos x + 2 = 0$  in the interval  $[0, 5\pi]$  is \_\_\_\_\_

(A) 5

(B) 0

(C) 6

(D) 10

33) For equilateral  $\Delta ABC$  if  $b = 3$  then circum radius  $R = \underline{\hspace{2cm}}$

- (A) 3
- (B)  $3\sqrt{3}$
- (C)  $\sqrt{3}$
- (D) 9

34) The sum of first fifteen terms of A.P. 16, 12, 8, 4 - - - - is  
 $\underline{\hspace{2cm}}$

- (A) -240
- (B) -195
- (C) -180
- (D) -345

35) The difference of 12<sup>th</sup> and 21<sup>st</sup> terms of sequence  $\left\{ \frac{n - (-1)^n}{11} \right\}$

is  $\underline{\hspace{2cm}}$

- (A) 1
- (B) 0
- (C) -1
- (D) 3

36)  $\sum_{i=6}^{10} (2i^2 + 3) = \underline{\hspace{2cm}}$

- (A) 675
- (B) 890
- (C) 435
- (D) 265

37) For G.P.  $t_2 = 7$ ,  $t_5 = -56$  then  $t_6 = \text{_____}$ .

- (A) 112
- (B) 56
- (C) -48
- (D) 128

38) For positive real numbers  $x, y, z$

- $$(x+y)(y+z)(z+x) \text{ _____}$$
- (A)  $< xyz$
  - (B)  $\geq 4xyz$
  - (C)  $\leq 4(x+y+z)$
  - (D)  $\geq 8xyz$

39) For G.P.  $4, 4^2, 4^3, \dots$  the sum of first  $n$  terms is 340 then value of  $n$  is  $\text{_____}$ .

- (A) 2
- (B) 1
- (C) 4
- (D) 6

40) Distance from point to focus, which is on Parabola  $y^2 = 8x$  is 6 then find the co-ordinates of points.

- (A)  $(6, \pm 4\sqrt{3})$
- (B)  $(2, \pm 4\sqrt{2})$
- (C)  $(1, \pm 2\sqrt{2})$
- (D)  $(4, \pm 4\sqrt{2})$

- 41) Find radius of circle passing through foci of ellipse

Rough Work

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 \text{ and having centre } (0, 5)$$

- (A)  $\sqrt{32}$
- (B) 25
- (C) 16
- (D) 9

- 42) Equation for the circle drawn on a diagonal of the rectangle as it's diameter whose sides are  $x = 2$ ,  $x = -4$ ,  $y = 3$ ,  $y = -6$

- (A)  $x^2 + y^2 + 2x - 3y + 26 = 0$
- (B)  $x^2 + y^2 - 2x - 3y + 26 = 0$
- (C)  $x^2 + y^2 - 2x + 3y - 26 = 0$
- (D)  $x^2 + y^2 + 2x + 3y - 26 = 0$

- 43) The product of eccentricities of hyperbolas  $x^2 - y^2 = 4$  and  $x^2 - y^2 = 9$  is \_\_\_\_\_.

- |       |       |
|-------|-------|
| (A) 4 | (B) 9 |
| (C) 2 | (D) 6 |

- 44) The equations  $x = 2 \sin^2 t$  and  $y = 2 \cos t$ ,  $t \in \mathbb{R}$  represents the graph of \_\_\_\_\_ curve.

- (A) Ellipse
- (B) Parabola
- (C) Circle
- (D) Hyperbola

45) The directions of vectors  $\bar{x} = (e, \pi, -e)$  and  $\bar{y} = (\pi, \frac{\pi^2}{e}, -\pi)$

are \_\_\_\_\_. Where  $e \in \mathbb{R}^+$

- (A) opposite
- (B) same
- (C) different
- (D) not defined

46) The point on the  $y$ -axis at distance  $\sqrt{10}$  from the point  $(3, 2, 1)$  is \_\_\_\_\_.

- (A)  $(0, 10, 0)$
- (B)  $(0, 2, 0)$
- (C)  $(0, 1, 0)$
- (D)  $(0, 3, 0)$

47) If  $A(3, -1, 5)$  and  $B(7, -1, 2)$  then  $|\overrightarrow{AB}| = \text{_____}$ .

- (A) 5
- (B) -5
- (C) 7
- (D) 1

**48)** If  $y(1, 2) = x(3, 1) + (1, 3)$  then  $x = \underline{\hspace{2cm}}$  and  
 $y = \underline{\hspace{2cm}}$ .

Rough Work

(A)  $\frac{1}{5}, \frac{8}{5}$

(B)  $5, 4$

(C)  $-\frac{1}{7}, \frac{5}{7}$

(D)  $-\frac{1}{5}, \frac{7}{5}$

**49)** A(1, 2, 4), B(1, 2, 0), C(1, 5, 0) are the vertices of  $\underline{\hspace{2cm}}$  triangle.

(A) isosceles

(B) equilateral

(C) right angled

(D) isosceles right angled

**50)**  $\lim_{x \rightarrow (-2)} \frac{x^3 - 3x + 2}{x^3 + x^2 + 4x + 12} = \underline{\hspace{2cm}}$

(A)  $\frac{4}{3}$

(B)  $\frac{3}{4}$

(C) 1

(D) 0

**050 (E)**

(MARCH/APRIL, 2015)

**(Part - B)***Time : 2 Hours/**/Maximum Marks : 50***Instructions :**

- 1) Write in a clear legible handwriting.
  - 2) There are three sections in Part - B of the question paper and total 1 to 18 questions are there.
  - 3) All the questions are compulsory. Internal options are given.
  - 4) The numbers at right side represent the marks of the question.
  - 5) Start new section on new page.
  - 6) Maintain sequence.
- 

**SECTION - A**

- Answer the following (1 to 8) questions as directed in the question. Each question carry 2 marks. [16]

- 1) Using principle of mathematical induction prove that  
 $3^{2n} - 8n - 1$  is divisible by 64;  $n \in \mathbb{N}$
- 2) Using P.M.I. prove that  $2 + 4 + 6 + 8 + \dots + 2n = n(n + 1)$ ,  $n \in \mathbb{N}$
- 3) Find the complex number Z whose modulus is 4 and principle argument is  $\frac{\pi}{3}$ .
- 4) Solve:  $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$ ,  $x \in \mathbb{C}$



5) If  $\sin \theta = \frac{7}{10}$  ( $0 < \theta < \pi/2$ ) then find value of

$$\frac{\cos(\pi-\theta)}{\cos(\pi+\theta)}, \frac{\sec(\pi+\theta)}{\sec(-\pi+\theta)}, \frac{\sec(2\pi+\theta)}{\cos(3\pi-\theta)}$$

OR

$$\text{Prove: } \sqrt{2} \sin 10^\circ + \sqrt{3} \cos 35^\circ = \cos 35^\circ + 2 \sin 25^\circ$$

6) Find solution set of  $4 \cos^2 \theta - 8 \sin \theta + 1 = 0$

7) Find co-ordinates of foci and equations of directrix of hyperbola

$$16x^2 - 9y^2 = -144.$$

OR

If one of the vertex of focal chord of parabola  $y^2 = 8x$  is  $\left(\frac{1}{2}, -2\right)$  then find other vertex.

8) Using first principle find derivative of  $\sqrt{\tan x}$ .

### SECTION - B

■ Answer the following questions (9 to 14) as directed in the question. Each question carry 3 marks. [18]

9) Using P.M.I. prove that

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}, (n \in \mathbb{N})$$

OR

Using P.M.I. prove that  ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n, (n \in \mathbb{N})$

10) If  $\cos(\alpha + \beta) = \frac{4}{5}$ ,  $\sin(\alpha - \beta) = \frac{5}{13}$  then find value of  $\cot 2\alpha$

$$\left( 0 < \alpha < \frac{\pi}{4}, 0 < \beta < \frac{\pi}{4} \right)$$

11) For  $\Delta ABC$ ,  $\frac{\sin C}{\sin A} = \frac{\sin(B-C)}{\sin(A-B)}$  then prove that  $c^2, b^2, a^2$  are in A.P.

OR

If the length of the two sides of a triangle are the roots of the equation  $x^2 - 2\sqrt{3}x + 2 = 0$  and if the included angle between them has measure  $\frac{\pi}{3}$  then find perimeter of the triangle.

12) Find the equation of the circle passing through the points (1, 4) and (5, 6) and having centre on the line  $x + 4y = 16$ . Also find area of circle.

13) Find the equation of the set of points P whose sum of the distances from the points A(4, 0, 0) and B(-4, 0, 0) is 10.

14) If  $f(x) = \frac{4x+7\sin x}{15x-8\cos x}$  then find  $f'(x)$  hence deduce value of  $f'(0)$ .

### SECTION - C

■ Answer the following questions (Nos. 15 to 18) as directed. Each question carry 4 marks. [16]

15) Find  $n$ , if the ratio of the fourth term to the fourth term from the end in the

expansion of  $\left(\sqrt[4]{3} + \frac{1}{\sqrt[4]{5}}\right)^n$  is 15:1.

16) Prove:  $\tan 142\frac{1}{2}^\circ = (\sqrt{2} - \sqrt{3}) \cot \frac{\pi}{8}$ .

OR

If  $A + B + C = \pi$  then prove

$$\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = 8 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}.$$

- 17) Sum of six consecutive terms of an A.P. is 48 and the product of the first and the last numbers is 39 find these numbers.

18) Obtain  $\lim_{x \rightarrow 0} \frac{\tan 5x - \tan 3x - \tan 2x}{30x^3}$ .

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